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Research article

Empirical Model for Moon Sighting

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Keywords: Empirical Model, Moon Sighting, Crescent Visibility, Lunar Phases, Waxing crescent

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An empirical model is developed to precisely identify the new crescent visibility of each Hijri month. The model is based on the actual real measurements of the moon motion. The model used the sinusoidal behaviour to model the relative motion of the lunar with respect to the earth. The model stability is achieved by the exact matching between the sine wave and the phases of the moon crescent, with respect to an observer from the earth. The accuracy of the model is granted by using the most accurate measurements for the motion of the moon. The model is applied to 18 Hijri months, from Shaban 1443 H to Muharam 1445 H. The moon sighting of these 18 months are obtained and compared with three references, the Um-Alqura calendar, Al-Tawqifat Alelhamih book and the moon visibility curves. The comparisons are discussed and concluded the reliability of the present model.

1. INTRODUCTION

The history of studying the lunar sciences started early, not only from the great encyclopedic scientist Ibn Al-Haitham (354-430 H or 965-1040 G) but also from the time of Aristo (384-322 BG), Ptolemy (87-150 G) and Eqlides (300 BG). The early works were based mainly on recording observations (95 dated observations, rewritten by O. Pedersen 2011[1]) and concluding their theories. In the front of his article, Carman 2019[2] mentioned Ptolemy's 36 observations, 4 observations of Theon of Alexandria, and the earliest lunar eclipse. But Ibn Al-Haitham corrected many of these theories such as the error of Ptolemy's assumption of the earth's stagnation and the rotation of other plants including the sun around it. Lippincott and Ringree 1987,[3] translated the codex representing three works of Ibn Al-Haitham. The codex was among many other codices assembled by Federico da Montefeltro, Duke of Urbino from 1474 to 1482. The later gave the codex to an Italian scholar to translate it from Arabic to Latin. The bilingual Arabic and Latin manuscript now in the Vatican Library (Urb. lat. 1384; Pls 14, 15, I6a), according to [3]. The codex, of Ibn Al-Haitham, identified 28 mansions, the description of the solar system and the definition of the zodiacal signs for each planet. He ended the codex by some judgement to the orbital trajectories of the solar system, according to the translation of [3], Ibn Al-Haitham wrote:

> The Sun = the Amir, Venus = the Wazir, the Scribe = the messenger, the Moon = peerless dancer, Jupiter = the judge, the Killer = the jailer, Mars = the policeman & executioner.

Up to the author knowledge, the translation of [3] failed to explain the first two names. Instead, the translation should be "The Sun is the Prince, and Venus is the Minster". But the most important definition, according to the present study, is the given description to the moon "peerless dancer". It is a good remark to how difficult is the moon motion. The present study eliminates this difficulty by focusing on the relative motion of the moon with respect to the earth rather than the absolute moon motion. This relative study serves exactly the purpose of the study as the moon sighting always done by an observer from the earth.

Many researches were interested in the Lunar characteristics to reach and landing on it. Such as identifying the lunar attitude by Lis Brack-Bernsen and Hermann Hunger (2006)[4] and Carman 2019.[2] But the efforts for moon sighting are less by researchers, from the author's point of view. One of the excellent attempts to identify the start of Hijri months is the early Egyptian book which was written by Mohamed Mokhtar in 1893.[5] This book includes the start of every Hijri month from the first year 1 H to 1500 H. So the book is used with other two references to compare and validate the present results. Therefore, this work is considered as an attempt to develop an empirical model valid for moon sighting and identifying the start of Hijri months to serve the whole Islamic Nations by only one calendar.

2. MOON SIGHTING

Mainly there exist two different approaches; the computational approach which was based on some mathematical models and the visual approach which was based on optical vision aids by using different telescopes, Tesla 2016.[6] As almost all researches were not targeting the moon sighting, waxing crescent, it becomes Muslim duty to give more attention for this important Islamic religious (Shariah) part. In this aspect the present study doesn't focus on the ab-

solute lunar motion but on the relative motion of the moon with respect to an observer on the earth.

The lunar motion characteristics with respect to the earth are:

1. 12 revolutions around the earth per Hijri year.[7]
2. 1 revolution takes 29.530588 days/month.[7]
3. The Hijri year is 354.36056 days.[7]
4. Full Moon was observed on 30/11/2020 at 10:29:55.[7]
5. Full Moon was observed on 30/12/2020 at 04:28:18.[7]

The value 29.530588 days/month which is given in the point 2 above needs to be confirmed. There is one verse in the Quran says that

> وَلَبِثُوا فِي كَهْفِهِمْ ثَلَاثَ مِائَةٍ سِنِينَ وَازْدَادُوا تِسْعًا (25)سورة الكهف

The meaning of this verse gives a very important relation between the solar year SY and the lunar year LY. In short, the verse defines the following relation:

$$300\ \text{SY} = 309\ \text{LY}$$

By using this fact, a new value to the number of days per Hijri month can be found:

$$\frac{300\ \text{SY} * 365.2425\ \text{days}}{309\ \text{LY} * 12\ \text{months}} = 29.55036408\ \text{days/month}$$

Recently, the solar year was counted as only 365.2425 days. On the other hand, the solar year was counted as just 365 days before the Julian calendar. Therefore the calculation should take the same figure which used in the time of the story before the updating to the exact year days.

$$\frac{300\ \text{SY} * 365\ \text{days}}{309\ \text{LY} * 12\ \text{months}} = 29.53074434\ \text{days/month}$$

This value confirms the measured value given in point 2 which is used in the rest of calculations.

3. THE MATHEMATICAL MODEL FOR MOON SIGHTING

The required model should reflect the phases of the moon with respect to an observer from the earth. The sinusoidal behavior of the moon phases gives the fundamental form of the model as follows:

$$A = D\ |\sin(wt + \theta)| \tag{1}$$

$A =$ the thickness of the luminous part of the moon.
$D =$ the moon diameter.
$w =$ the angular velocity of the moon, rad/day.
$t =$ the time in days.
$\theta =$ the shift angle from the sighting time 18:00:00.

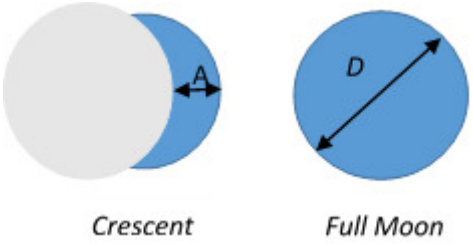

Figure 1. Luminus Crescent.

In calculations the ratio of *A/D* is the required to identify the moon sighting, in which this value will change sinusoidal through each month, from zero in the beginning, to maximum of unity in the middle to zero again in the end of the month.

$$\frac{A}{D} = |\sin(wt + \theta)| \tag{2}$$

The angular velocity *w* can be calculated using Eq. 3:

$$w = \frac{2\,\pi}{2 * 29.530588} = 0.106384358\ rad/day \tag{3}$$

Calculations of θ is based on full moon sighting given in the point 5 mentioned above, as the last recorded measurements on [7]. As the full moon was observed on that date and time, it can be taken as a reference value (t=0, on 29/12/2020) as follows:

$$\frac{A}{D} = 1, \quad where \quad t = \frac{10 + \frac{28}{60} + \frac{18}{3600}}{24}\ days \tag{4}$$

Noting that the study consider the calculations at average reference time 18:00:00.

$$\theta = \frac{\pi}{2} - 0.106384358 * \frac{10 + \frac{28}{60} + \frac{18}{3600}}{24} = 1.524387763\ rad \tag{5}$$

Now the model is ready and based on the most available accurate measured values as follows:

$$\frac{A}{D} = |\sin(0.106384358\,t + 1.524387763)| \tag{6}$$

The model validation is verified by calculating the previous full moon indicated in point 4 above. The given date and time is substituted in the model and the moon ratio, *A/D* is calculated. If the obtained ratio is equal 1, then the model is verified.

$$\frac{A}{D} = |\sin(w * -29.31255787 + \theta)| = 0.999730368$$

The validation test shows that the error is within $3*10^{-4}$.

4. MODEL RESULTS

The calculated moon ratio, *A/D*, defines the phases of the moon, so its value ranges as follows:

$$0 \le \frac{A}{D} \le 1 \tag{7}$$

The full moon is identified when the moon ratio equals 1 and happen in the middle of a month. The zero or very small values are expected at the end and beginning of a month. The main two differences between these two cases is that, for the first difference, at the end of the month the moon ratio is decreasing whenever in the month beginning the ratio is increasing. The second main difference is that, the last crescent (Waning crescent) appears near Fajr (sun rise) but the first crescent (Waxing crescent) appears near Maghreb (sun set). The first difference is important in calculations but the second difference is essential in both calculations and naked eye visibility. To watch how much the moon ratios of the first crescent are small the following figure 2 is introduced.

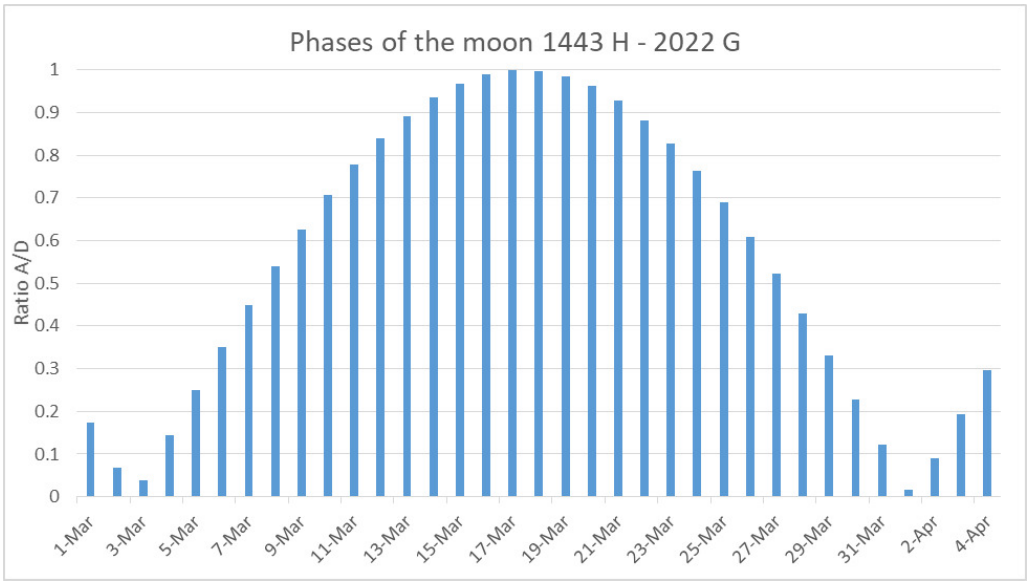

Figure 3a. The Moon's phases of the March 2022.

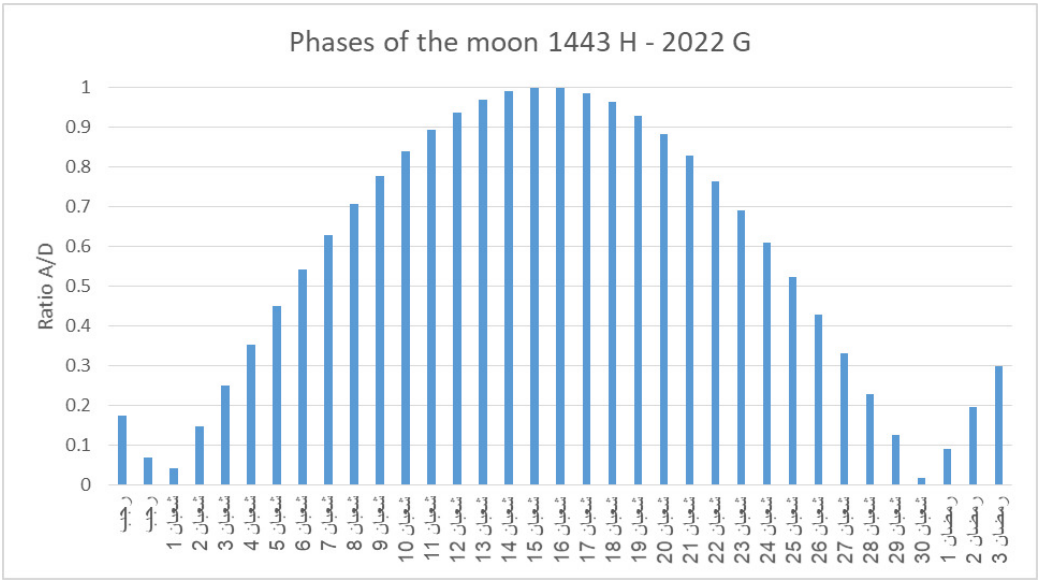

Figure 3b. The Moon's phases of the Shaban 1443 H.

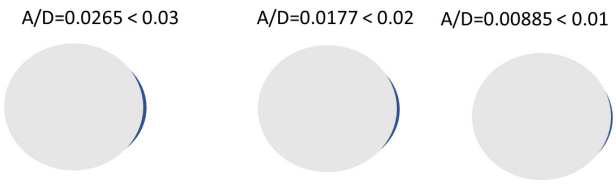

Figure 2. The new luminous crescent ratios (in blue).

The ratios less than 0.01 are difficult to be observed by naked eye in which optical instrumentations are welcomed.

The model is used to define the first Hijri month (Shaban) from the date of March 1, 2022. The full crescent sighting and phases for the March are presented in figure 3a. Figure 3b presents the results of Moon phases for Shaban 1443 H.

The most important part is the start of each month. So the next figures are focused on the months' starts.

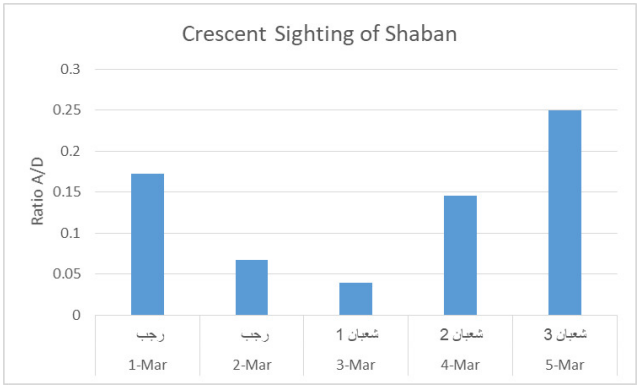

Figure 4. The model's crescent visibility of Shaban 1443 H.

By comparing the crescent in March 3 with both crescents in March 2 and 4, it is proved that the crescent of March 3 is increasing one (the second rate of change of A/D is positive), see figure 6. So it represents the beginning of Shaban 1443 H. This result agrees the UM-Alqura calendar[8] and the well-known visibility curves.[9] But The Al-Tawqifat Alelhamih[5] gave 4-Mar instead of 3-Mar.

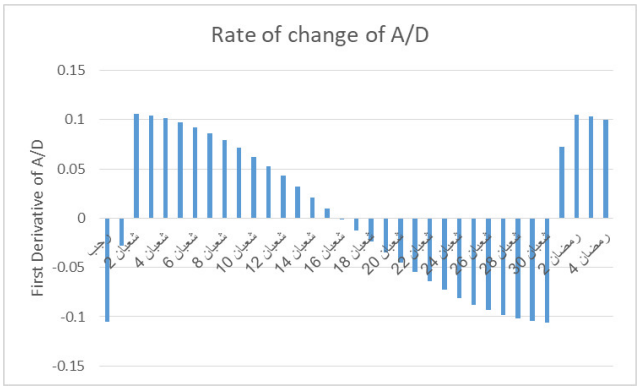

Figure 5. The rate of change of the Crescent Ratio A/D.

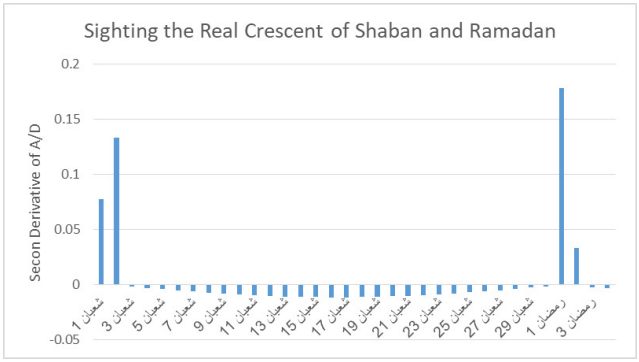

Figure 6. Positive Values of $(A/D)^{\backslash\backslash}$ Proves The Exact Crescent Sighting.

On the other hand to detect exact new crescent sighting, the second derivative of A/D is used. Figure 6 shows the positive values of $(A/D)^{\backslash\backslash}$ with the new crescent.

The crescent of Ramadan 1443 H is shown in figure 7.

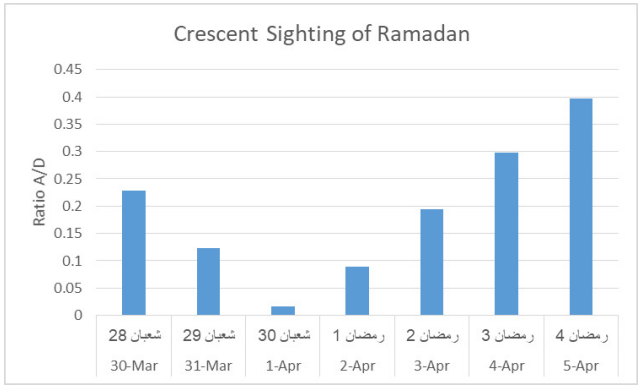

Figure 7. The model's crescent visibility of Ramadan 1443 H.

By comparing the crescent in April 1 with both crescents in March 31 and April 2, it is proved in figure 8 that the crescent of April 1 is decreasing. So April 2, as the crescent ratio is increasing, represents the beginning of Ramadan 1443 H, see figure 8. This result agrees with Al-Tawqifat Alelhamih calendar[5] and the well-known visibility curves.[9] But the UM-Alqura calendar gives the visibility of Ramadan crescent in 1-Apr instead of 2-Apr.

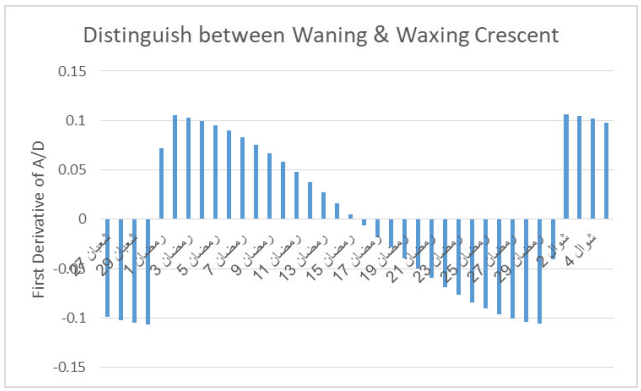

Figure 8. The Rate of change of A/D.

For proofing the exact crescent sighting of Ramadan, figure 9 is presented below. The figure shows the positive value of $\frac{d^2}{dt^2}\left(\frac{A}{D}\right)$ which identifies the crescent visibility.

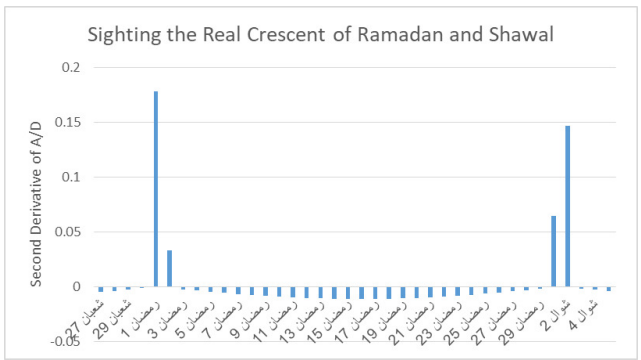

Figure 9. The Positive values of $(A/D)^{\backslash\backslash}$ Represents The Exact Start of The Month.

The first crescent of Shawal is illustrated in figure 10.

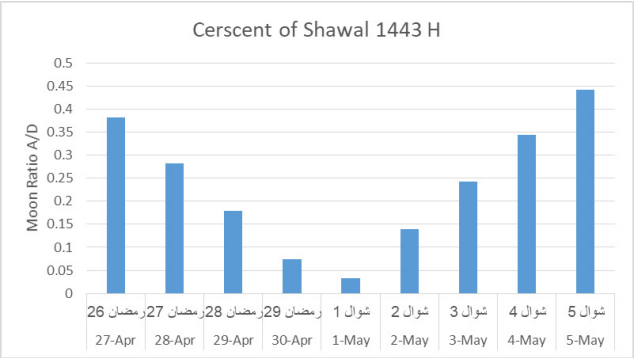

Figure 10. The model's crescent visibility of Shawal 1443 H.

By comparing the crescent in May 1 with both crescents in April 30 and May 2, it is proved that the crescent of May 1 is increasing. So it represents the beginning of Shawal 1443 H. This result contradicts the Al-Tawqifat Alelhamih.[5] But it agrees with the UM-Alqura calendar[8] and the visibility curves.[9]

The crescent of Dul-Qaidah is given below.

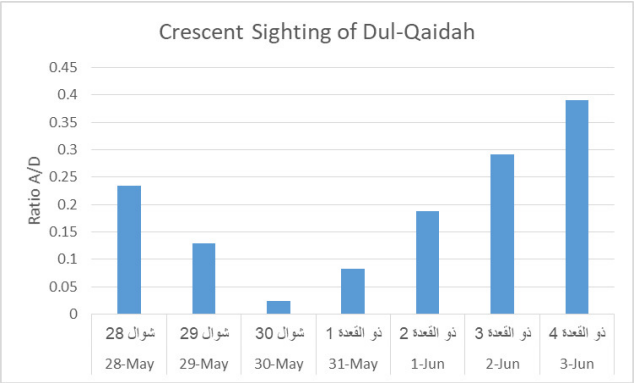

Figure 11. The model's crescent visibility of Dul-Qaidah 1443

By comparing the crescent in May 30 with both crescents in May 29 and May 31, it is proved that the crescent of May 30 is decreasing one. So May 31 represents the beginning of Dul-Qaidah 1443 H. This result agrees with both the Al-Tawqifat Alelhamih[5] and the visibility curves.[9] But the UM-Alqura calendar[8] gives the visibility of Dul-Qaidah's crescent in 30-May instead of 31-May.

The summary of all results, from Shaban 1443 to Muharam 1445 H are given in table 1.

5. DISCUSSIONS AND CONCLUSIONS

Results, of the present work and 3 references, for the month start date over 18 different months are presented in table 1. The Um-Alqura calendar [8] and Al-Tawqifat Alelhamih[5] name the first day of any Hijri month by the second day of the crescent visibility. For example if the crescent of Shaban was seen on March 3, then both [5] and [8] calendars give that 1 Shaban is March 4. But from the author's point of view, the start of the day and consequently the start of the month from the time of new crescent visibility is confirmed and not from the second day. Accordingly, the comparison between all results is highlighted as shown in table 2.

The odd results are highlighted in red in table 2. The present model indicates no odd results. The visibility curves[9] shows also no odd results. The calendar of Um-Alqura[8] has 2 odd results and Al-Tawqifat Alelhamih[5] has 4 odd results. On the other hand, all 4 techniques are agreed on 8 different months, highlighted in green. Hence the start of these 8 months are fully obtained with certainty almost 100%. The case of 3 techniques agree against only one technique disagree is happened in 6 months and highlighted in blue. This case confirms the month start within the certainty of 75%. But the case of 2 techniques agree against the other 2 techniques, which reflects a certainty of 50%, is happened in 4 months and highlighted in yellow. This last case needs more evidences to confirm one start of the month.

Concerning the last 3 months, highlighted in yellow, of the equal certainty 50%: The two references, [8] and [9], prove the visibility of the new crescent one day before the present model and Al-Tawqifat Alelhamih.[5] The proofs of the three months of 1444 H, Shaban, Shawal and Dul-Hejah, are presented in Figures 12-14.

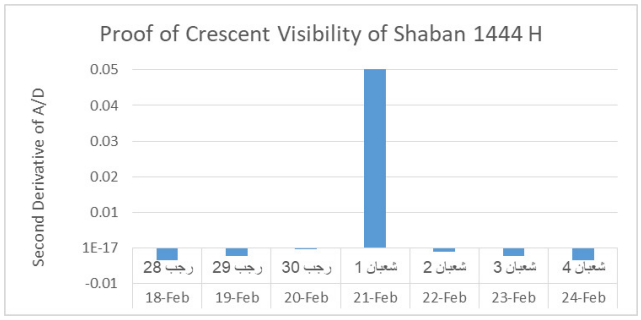

Figure 12. The Present Model Crescent Visibility of Shaban 1444 H.

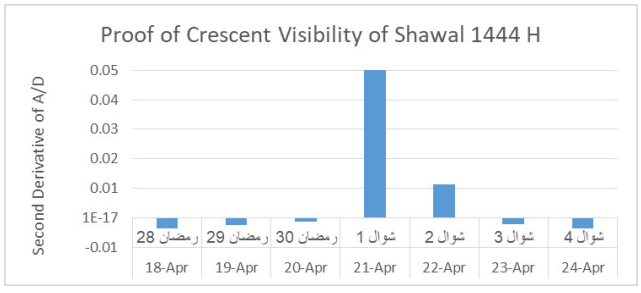

Figure 13. The Present Model Crescent Visibility of Shawal 1444 H.

Table 1. The results of the present model Vs 3 references.

Hijri Months	Starting date of Hijri Months			
	Present Model	Um-Alqura Calendar[7]	Visibility Curves[8]	Al-Tawqifat Alelhamih[5]
Shaban 1443 H	3-Mar-2022	4-Mar	3-Mar	5-Mar
Ramadan	2-Apr	2-Apr	2-Apr	3-Apr
Shawal	1-May	2-May	1-May	3-May
Dul-Qaidah	31-May	31-May	31-May	1-June
Dul-Hejah	29-Jun	30-Jun	29-Jun	1-Jul
Muharam 1444 H	29-Jul	30-Jul	29-Jul	30-Jul
Safar	27-Aug	28-Aug	27-Aug	29-Aug
Rabi 1	26-Sep	27-Sep	26-Sep	27-Sep
Rabi 2	25-Oct	26-Oct	26-Oct	27-Oct
Jumada 1	24-Nov	25-Nov	24-Nov	25-Nov
Jumada 2	24-Dec	25-Dec	24-Dec	25-Dec
Rajab	22-Jan-2023	23-Jan	22-Jan	23-Jan
Shaban	21-Feb	21-Feb	20-Feb	22-Feb
Ramadan	22-Mar	23-Mar	22-Mar	23-Mar
Shawal	21-Apr	21-Apr	20-Apr	22-Apr
Dul-Qaidah	20-May	21-May	20-May	21-May
Dul-Hejah	19-Jun	19-Jun	18-Jun	20-Jun
Muharam 1445 H	18-Jul	19-Jul	18-Jul	19-Jul

Table 2. The results comparison of the present model Vs 3 references.

Hijri Months	Starting date of Hijri Months			
	Present Model	Um-Alqura Calendar[8]	Visibility Curves[9]	Al-Tawqifat Alelhamih[5]
Shaban	3-Mar-2022	4-Mar	3-Mar	5-Mar
Ramadan	2-Apr	2-Apr	2-Apr	3-Apr
Shawal	1-May	2-May	1-May	3-May
Dul-Qaidah	31-May	31-May	31-May	1-June
Dul-Hejah	29-Jun	30-Jun	29-Jun	1-Jul
Muharam-44	29-Jul	30-Jul	29-Jul	30-Jul
Safar	27-Aug	28-Aug	27-Aug	29-Aug
Rabi 1	26-Sep	27-Sep	26-Sep	27-Sep
Rabi 2	25-Oct	26-Oct	26-Oct	27-Oct
Jumada 1	24-Nov	25-Nov	24-Nov	25-Nov
Jumada 2	24-Dec	25-Dec	24-Dec	25-Dec
Rajab	22-Jan-2023	23-Jan	22-Jan	23-Jan
Shaban	21-Feb	21-Feb	20-Feb	22-Feb
Ramadan	22-Mar	23-Mar	22-Mar	23-Mar
Shawal	21-Apr	21-Apr	20-Apr	22-Apr
Dul-Qaidah	20-May	21-May	20-May	21-May
Dul-Hejah	19-Jun	19-Jun	18-Jun	20-Jun
Muharam-45	18-Jul	19-Jul	18-Jul	19-Jul

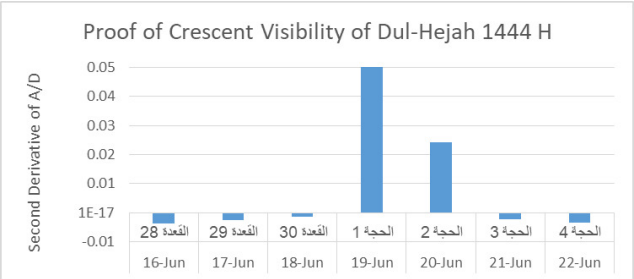

Figure 14. The Present Model Crescent Visibility of Dul-Hejah 1444 H.

The same case appears again in the two odds of the calendar of Um-Alqura: The calendar proves the visibility of crescent one day before the new crescent. The proof of the new crescent of Ramadan 1443 H is presented in figure 9. Figure 15 shows the exact new crescent of Dul-Qaidah 1443 H.

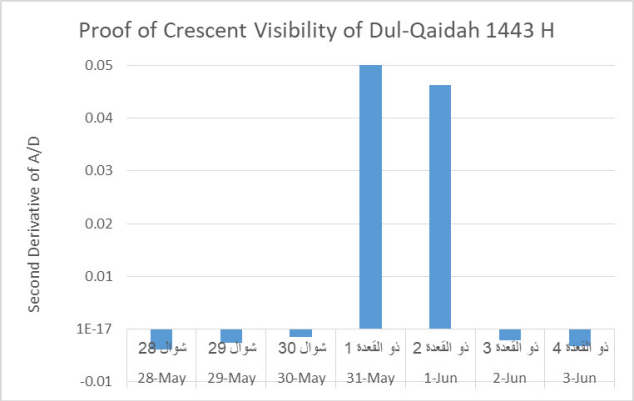

Figure 15. The Present Model Crescent Visibility of Dul-Qaidah 1443 H.

The last equal certainty result of Rabi 2 1444 H, the two references, [5] and [9], prove the crescent visibility one day after the new crescent given by the present work and [8]. The present model calculations in figure 16 proves the existence of the new crescent and its visibility in the October 25, not 26, as also confirmed by the Um-Alqura calendar.

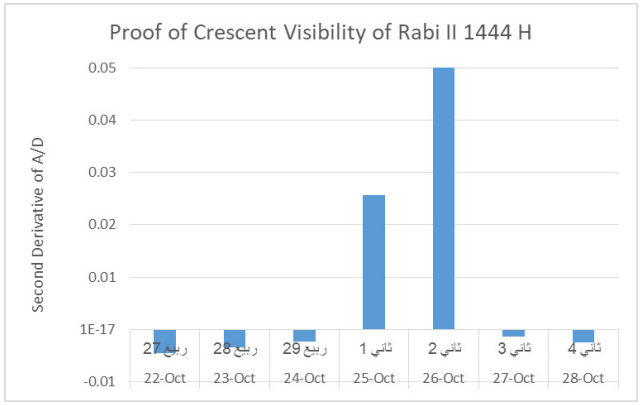

Figure 16. The Present Model Crescent Visibility of Rabi II 1444 H.

Therefore, the model proofs the reliability and accuracy to obtain the moon sighting/phases, and crescent visibility. Also, the model is able to create the Hijri calendar in highest accuracy. The crescent sighting depends on the three values which are the moon ratio A/D and its rate of changes up to the second derivative. The use of these three values granted the highest accuracy of identifying the exact months' starts.

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ABBREVIATION

LY Lunar Year
SY Solar Year

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